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Status of nonsupersymmetric grand unified theories

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Abstract : Prospects of certain nonsupersymmetric grand unified theories consistent with the CERN-LEP data and experimental lower limit on proton lifetime are reviewed. Models with Higgs scalars in the grand desert require additional finetuning of parameters. More natural candidates are $SO(10)$, $E(6)$ and $SU(16)$ *etc* with a left-right gauge group as intermediate symmetry where no additional finetuning is needed. Such models have potentialities to explain small neutrino masses and mixings relevant for neutrino-oscillation searches. Unification of Yukawa couplings for the third generation fermions at the scale ($M_1 \simeq 10^9 - 10^{11}$ GeV) is achieved in the nonSUSY two-Higgs doublet standard model with $SU(2)_L \times SU(2)_R \times SU(4)_C$ intermediate breaking.

Keywords : GUT, neutrino oscillation, Yukawa couplings

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1. Introduction

Unification of strong, weak, and electromagnetic interactions were originally suggested in the framework of non-supersymmetric (nonSUSY) grand unified theories (GUTs) [1–3] and has been generalised in the context of supersymmetric (SUSY) GUTs. Unlike supersymmetric theories, where there is a natural solution to the gauge hierarchy problem, the nonsupersymmetric GUTs rely upon extended survival hypothesis consistent with the minimal finetuning of parameters to maintain the gauge hierarchy [4]. On the other hand, where as proton decay has a neat prediction in nonSUSY GUTs, the predictive power in SUSY GUTs are seriously affected by the presence of additional contributions through higher dimentional operators. Unless these additional terms are suppressed, by imposing new symmetries, or otherwise, proton loses its stability in SUSY GUTs. In his review, we discuss nonSUSY GUTs with single intermediate gauge symmetrices which are in agreement with the existing data from CERN-LEP and the proton life time measurements on $p \to e^+\pi^0$. Such models are also consistent with the minimal finetuning of parameters. We also discuss nonSUSY models with Higgs scalars substantially lighter than the GUT scale. $t-b-\tau$ Yukawa unification at the intermediate scale ($M_I \simeq 10^9 - 10^{11}$ GeV) has been recently demonstrated in nonSUSY $SO(10)$ [5], with $SU(2)_L \times SU(2)_R \times SU(4)_C$ ($g_{2L} \neq g_{2R}$) intermediate gauge symmetry. Prospects for neutrino masses and mixings are briefly mentioned.

2. Modifications of the grand desert models

2.1. *Difficulties with the grand desert models*:

In the minimal $SU(5)$, all standard fermions of one generation are unified into two different representations : $\bar{5}+10$. Spontaneous symmetry breaking of $SU(5)$ to the standard model (SM $= SU(2)_L \times U(1)_Y \times SU(3)_C \equiv G_{213}$), and of SM to the low-energy group $U(1)_{em} \times SU(3)_c$ take place by $\underline{24}$ and $\underline{5}$ of $SU(5)$. The colour triplet in $\underline{5}$ acquires GUT-scale mass and the weak doublet is kept at the electroweak scale by minimal finetuning of parameters. The spectrum of fermions, Higgs scalars and gauge bosons in $SU(5)$ or single step breaking of other GUTs, below the GUT scale, is that of the nonSUSY standard model.

The three gauge couplings of the SM are now known from the CERN-LEP data at $\mu = M_Z$,

$$\alpha_1(M_Z) = 0.01688 \pm 0.00004,$$
$$\alpha_2(M_Z) = 0.03322 \pm 0.00025,$$
$$\alpha_3(M_Z) = 0.118 \pm 0.007. \tag{1}$$

The renormalization group equations (RGEs) below the GUT scale (M_U) are [6,7]

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(M_U)} + \frac{b_i}{2\pi}\ln\frac{M_U}{\mu} + \theta_i - \Delta_i, \tag{2}$$

where $\theta_i(\Delta_i)$ takes into account the two-loop (threshold and other) effects. At two-loop level, the two electroweak gauge couplings meet near $\mu \simeq 10^{13}$ GeV. Using the grand unification condition

$$\alpha_1(M_U) = \alpha_2(M_U) = \alpha_3(M_U) \tag{3}$$

and running $\alpha_3(\mu)$ from $M_U \simeq 10^{13}$ GeV to M_Z through RGE gives

$$\alpha_3(M_Z) \equiv \alpha_s(M_Z) \simeq 0.073 \tag{4}$$

which is too low as compared to CERN-LEP data. Further, the proton lifetime with such low unification mass, is nearly 8 orders of magnitude less than the experimental lower limit for $p \to e^+\pi^0$ mode.

During seventies, the value of electroweak mixing angle measured from neutral current data turned out to be $\sin^2\theta_W = 0.21$ which along with $\alpha_s(M_Z) = 0.118$ gives $M_U \simeq 10^{14.5}$ GeV. As we know, the present data gives $\sin^2\theta_W$ value nearly 10% larger and $M_U \geq 10^{15}$ GeV. Similarly, all non-SUSY GUTs like $SO(10)$, $SU(16)$ and E_6 *etc.* with a grand desert between $M_Z - M_U$ are ruled out by CERN-LEP data and proton lifetime measurements for $p \to e^+\pi^0$.

2.2. *Models with Higgs scalars in the grand desert* :

The presence of Higgs scalars in the grand desert with masses substantially smaller than M_U changes slopes of $\alpha_i^{-1}(\mu)$ (i = 1, 2, 3) and hence their point of intersection, M_U. Higgs scalars near M_Z or the TeV scale have been discussed in ref. [8] and in refs. [9,10] in $SU(5)$ with split multiplets. In such cases, more than one SM representations are to remain light which needs a number of additional finetuning of parameters. But it has been demonstrated in ref. [11] that only one standard model representation $\xi(3, 0, 8)$ with mass near $10^{10} - 10^{12}$ GeV is sufficient to guarantee unification of gauge couplings at $M_U \geq 10^{15}$ GeV. Such a Higgs scalar is contained $\underline{75} \subset SU(5)$, $210 \subset SO(10)$ and $651 \subset E_6$. The $SO(10)$ model predicts proton lifetime [11]

$$\tau_p(\mathrm{p} \rightarrow \mathrm{e}^+\pi^0) = 2.8 \times 10^{32 \pm 0.4 \pm 0.7^{+1.7}_{-2.6}} \text{ Yrs}, \tag{5}$$

which is accessible to the next generation experiments. In (5) the first, second and the third uncertainties are due to the input parameters, matrix element, and threshold effects. The third uncertainty depends on the type of GUTs. Due to gravitational corrections, there could be additional corrections to τ_p. The model has potentialities to account for neutrino masses necessary for solar neutrinos and hot dark matter of the universe. The unification of gauge couplings is shown in ref. [11]. In other models the predicted proton lifetime is larger [10].

3. GUTs with intermediate gauge symmetries

3.1. *Some intermediate symmetries* :

If there is a left-right gauge group beyond SM, such as $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$ ($\equiv G_{2213}$) or $SU(2)_L \times SU(2)_R \times SU(4)_C$ ($\equiv G_{224}$) with left-right discrete symmetry ($\equiv$ Parity, when $g_{2L} = g_{2R}$) or without it ($g_{2L} \neq g_{2R}$), it can explain the origins of parity (P) or CP violations. In addition $SU(4)_C$ [1], explains why quarks are different from leptons at low energies through quark-lepton unification at high scales. Further, because of the presence of a R.H. neutrino (N), the models provide neutrino masses over wide range of values *via* see-saw mechanism [12]. Both G_{2213} and G_{224} have rank five and the minimal grand unifying symmetry in which they can be embedded is $SO(10)$ [3]. Unlike $SU(5)$, where standard fermions are unified through two different representations, $\bar{5}$ and 10, they are unified in $SO(10)$ through the single spinorial representation, $\underline{16}$. Some other interesting GUTs which unify gauge mediating forces are E_6, $SO(18)$ and $SU(16)$ *etc.* Whereas the fermion representation of E_6 for every generation is $\underline{27}$, requring 12 new fermions than what are observed at low-energies, these in $SO(18)$ contain 8 families, each having a R.H.neutrino. Similarly, grand unification through $SU(16)$ needs mirror fermions for anomaly cancellation. For the sake of simplicity we will confine to $SO(10)$ grand unification with single intermediate gauge symmetry like G_{2213} or G_{224}, with or without parity.

3.2. *Decoupling of parity and $SU(2)_R$ breakings* :

To understand how $SO(10)$ breaking can yield left-right gauge groups with or without L-R discrete symmetry ($\equiv$ parity $\equiv$ P), it is necessary to follow D-parity and its breakings [13] in $SO(10)$.

The Higgs representations $\underline{45}$, $\underline{54}$, $\underline{210}$, $\underline{126}$ and $\underline{10}$ have the following decompositions under G_{224} :

$$\underline{45} = (1, 1, 15) + (3, 1, 1) + (1, 3, 1) + (2, 2, 6),$$

$$\underline{54} = (1, 1, 1) + () + (1, 1, 20) + (3, 3, 1) + (2, 2, 6),$$

$$\underline{210} = (1, 1, 1) + (1, 1, 15) + (3, 1, 15) + (1, 3, 15) + (2, 2, 10) + (2, 2, \overline{10}) + (2, 2, 6),$$

$$\underline{126} = (3, 1, 10) + (1, 3, \overline{10}) + (2, 2, 15) + (1, 1, 6),$$

$$10 = (2, 2, 1) + (1, 1, 6).$$

The G_{224}-singlet (1, 1, 1) in $\underline{54}$ ($\underline{210}$) is even (odd) under D-parity [13]. $SO(10)$ can break down to left-right gauge groups by preserving or breaking D-parity, but without breaking $SU(2)_L$ and $SU(2)_R$ gauge symmetries. The breakdown of D-parity is always accompanied by the breakdown of L-R discrete symmetry. If $SO(10)$ is broken by vacuum expectation values along G_{2213} (G_{224})-singlets, then D-parity and hence P are broken (left unbroken), if such singlets are odd (even) under D. Thus, when the Higgs representation $\underline{54}$ ($\underline{210}$) is used to break $SO(10)$, it yields G_{224D} (G_{224}) with (without) left-right discrete symmetry. Similarly (1, 1, 15) $\subset$ $\underline{210}$ contains a G_{2213}-singlet which is even under D whereas (1, 1, 15) $\subset$ $\underline{45}$ has a G_{2213}-singlet odd under D. These provide the possibility of having G_{2213} (G_{2213D}), if we break $SO(10)$ *via* $\underline{45}$ ($\underline{210}$). In order to obtain the standard gauge symmetry from the left right gauge groups and the low-energy symmetry $U(1)_{em} \times SU(3)_c$, while generating small Majorana neutrino masses by see-saw mechanism, the components $\Delta_R(1, 3, \overline{10}) \subset \underline{126}$ and $\Phi(2, 2, 1) \subset \underline{10}$ are utilised. The following interesting possibilities have been explored in non SUSY $SO(10)$ model [13–17] :

(A) $\quad SO(10) \xrightarrow[M_U]{\underline{54}} G_{224D} \xrightarrow[M_I]{\underline{126}} G_{213} \xrightarrow[M_Z]{\underline{10}} G_{13},$

(B) $\quad SO(10) \xrightarrow[M_U]{\underline{210}} G_{224} \xrightarrow[M_I]{\underline{126}} G_{213} \xrightarrow[M_Z]{\underline{10}} G_{13},$

(C) $\quad SO(10) \xrightarrow[M_U]{\underline{210}} G_{2213D} \xrightarrow[M_I]{\underline{126}} G_{213} \xrightarrow[M_Z]{\underline{10}} G_{13},$

(D) $\quad SO(10) \xrightarrow[M_U]{\underline{45}} G_{2213} \xrightarrow[M_I]{\underline{126}} G_{213} \xrightarrow[M_Z]{\underline{10}} G_{13},$

(E) $\quad SO(10) \xrightarrow{\underline{210}} G_{214} \xrightarrow{\underline{126}} G_{213} \xrightarrow{\underline{10}} G_{13}.$

In the cases (B) and (D), parity and $SU(2)_R$-breakings are decoupled. In the last case (E), the spontaneous symmetry breaking in the first step takes place by assigning VEV to the G_{214} ($\equiv SU(2)_L \times U(1)_R \times SU(4)_C$)-singlet component in (1, 3, 15) $\subset$ $\underline{210}$ [14]. The Models (A)–(E) are consistent with CERN–LEP measurements and lower limit on proton lifetime with unification of gauge couplings occurring between 10^{15} – $10^{16.5}$ GeV [15–17]. Such predictions are stable under loop corrections. Although threshold and gravitational corrections are smaller in models with G_{2213D} and G_{224D}, they are somewhat larger in the cases of G_{2213} and G_{224}. The Model (E) is ruled out at two-loop level [16], but allowed when threshold or gravitational effects are included [14,18].

3.3. Theorems for precise predictions:

One of the most interesting aspects of Model-(A) has been noted [19–21] to be the vanishing high-scale corrections on the predictions of $\sin^2\theta_W(M_Z)$ [19,20] and intermediate scale (M_I) [21] in all GUTs with G_{224D} intermediate symmetry, as summarized by three theorems:

Theorem 1 [Ref. 19] : In all grand unified theories where G_{224P} occurs as an intermediate gauge symmetry, all GUT-scale corrections to $\sin^2\theta_W(M_Z)$ vanish.

Theorem 2 [Ref. 20] : In all grand unified theories where G_{224P} occurs as an intermediate gauge symmetry, all high scale and multiloop corrections arising from $\mu \geq M_I - M_U$ are absent in $\sin^2\theta_W(M_Z)$.

Theorem 3 [Ref. 21] : In all grand unified theories where G_{224P} occurs as an intermediate gauge symmetry, all high scale corrections arising from $\mu \geq M_I - M_U$ are absent in the intermediate scale (M_I).

It is to be emphasized that these theorems hold good also in SUSY GUTs and superstring theories, if the latter permit G_{224P} as an intermediate gauge symmetry.

The corrections emerging from threshold and higher dimensional operator effects at the GUT scale, which are major sources of uncertainties in other models, exactly vanish in Model-(A). Threshold effects in nonSUSY GUTs near M_Z is much less compared to those in SUSY GUTs. Only significant corrections which contribute to the uncertainties in $\sin^2\theta_W$ and M_I are those at the intermediate scale. Thus, the stability in M_I necessarily leads to more precise predictions on neutrino masses as compared to other models [21].

3.4. Model predictions with intermediate scales:

In Table 1, we present predictions on intermediate scale (M_I) and unification mass (M_U) using CERN–LEP data as input for Models (A)–(E) in nonSUSY $SO(10)$. The uncertainties

Table-1. Predictions of intermediate scale and unification masses in non-SUSY $SO(10)$ with single intermediate gauge symmetries. The second (third) uncertainties in the masses are due to input parametres (threshold effects).

Model	M_I (GeV)	M_U (GeV)	α^{-1}
(A)	$10^{13.64 \pm 0.2 \pm 0.44}$	$10^{15.02 \pm 0.25 \pm 0.48}$	40.76 ± 0.16
(B)	$10^{10.70 \pm 0.2^{+2.68}_{-0.06}}$	$10^{16.26 \pm 0.25^{+0.13}_{-1.24}}$	46.35 ± 0.22
(C)	$10^{10.16 \pm 0.2 \pm 0.007}$	$10^{15.55 \pm 0.20 \pm 0.43}$	43.86 ± 0.18
(D)	$10^{9.08 \pm 0.2^{+0.30}_{-0.08}}$	$10^{16.42 \pm 0.22^{+0.18}_{-0.49}}$	46.12 ± 0.15
(E)	$10^{11 \pm 0.4 \pm 0.14}$	$10^{14.5 \pm 0.2 \pm 0.66}$	$44.32 \pm 0.35 \pm 2.1$

are due to threshold effects at the intermediate and GUT scales. Such intermediate scales are important for neutrino masses necessary to explain solar neutrino oscillations and ν_τ as hot component of dark matter of the universe [22,23].

With the unification masses given in Table 1, the predictions on proton lifetime are shown in Table 2, where the first, second, and third uncertainties are due to matrix elements, input data, and threshold effects. When higher dimensional operator effects at the

Table-2. Proton lifetime predictions for $p \to e^+\pi^0$ in non SUSY $SO(10)$ with single intermediate gauge symmetries. The first, second, and third uncertainties are due to matrix element, input parameters and threshold effects. For Models (A)–(D) degeneracy of components of a particular multiplet at the relevant scale has been imposed.

Model	τ_p(Yrs.)	Reference
(A)	$1.44 \times 10^{32.1 \pm 0.7 \pm 1.0 \pm 1.9}$	[17]
(B)	$1.44 \times 10^{37.4 \pm 0.7 \pm 1.0^{+0.5}_{-3.0}}$	[17]
(C)	$1.44 \times 10^{34.2 \pm 0.7 \pm 0.8 \pm 1.7}$	[17]
(D)	$1.44 \times 10^{37.7 \pm 0.7 \pm 0.9^{+0.7}_{-2.0}}$	[17]
(E)	$2.88 \times 10^{29.8 \pm 0.7 \pm 0.8 \pm 0.64}$	[23]

GUT scale are included [24], the uncertainties are likely to increase further. One advantage of computing uncertainties is that, although central value in τ_p is not within the reach in near future, the lowest allowed value for τ_p in Model-(B) overlaps with the SuperKamiokande limit.

4. Enhancement of $m_{\nu\tau}$ by radiative corrections

One of the interesting possibilities that has been explored recently is to have $m_{\nu\tau} \doteq 2 - 10$ eV, suggesting it as the hot component of dark matter (HDM) of the universe while other appropriate neutrino masses and mixings can account for $\nu_e - \nu_\mu$ oscillations in the interior of solar core. The most popular formula for neutrino masses is based upon see-saw mechanism [12],

$$m_{\nu_i} = C_{\nu_i} \frac{m_{q_i}^2}{M_N}; \qquad i = 1, 2, 3 \tag{6}$$

with ν_i $(i = 1, 2, 3) \equiv (\nu_e, \nu_\mu, \nu_\tau)$ and q_i $(i = 1, 2, 3) \equiv (u, c, t)$. With the canonical value $C_{\nu_i} = -1$, the formula (6) is valid at the high scale where lepton number breaks down spontaneously to generate heavy R.H.neutrino mass M_N assumed to be degenerate for three generations. Attempts have been made earlier [25] to obtain values of C_{ν_i} at low energies so as to compare the model predictions with the experimental data [25],

$$C_{\nu_e} = 0.05, \qquad C_{\nu_\mu} = 0.07, \qquad C_{\nu_\tau} = 0.18. \tag{7}$$

In an interesting observation [23], it has been demonstrated, more recently, that the observed largeness of the Yukawa coupling of the top-quark enhances C_{ν_τ}, significantly, compared to earlier estimations [25,26]. Such rediative corrections help in the interpretation of ν_τ as a hot dark matter candidate. In supersymmetric GUTs this approach leads to tan β-dependence in $m_{\nu_\tau} / m_{\nu_\mu}$ and m_{ν_τ} / m_{ν_e} [27]. For example, it has been shown that in nonSUSY Model-(B), the central values are [23]

$$C_{\nu_e} = 0.027, \quad C_{\nu_\mu} = 0.030, \quad C_{\nu_\tau} = 0.288 \tag{8}$$

showing more than 50% increase in C_{ν_τ} and hence m_{ν_τ} as compared to (7). Uncertainties in C_{ν_i} increase if the uncertainty in $\alpha_s(M_Z)$ is allowed to propagate below the Z-mass down to low energies.

5. Quark-lepton Yukawa unification at intermediate scales

In the presence of gauge bosons and Higgs scalars in unified theories, in addition to the unification of gauge mediated fundamental forces, unification of Yukawa forces mediated by Higgs scalars is a natural expectation. In the one-step breaking of SUSY GUTs, $b - \tau$ Yukawa unification with $h_b = h_\tau$ has been observed for smaller values of tan $\beta \simeq 2$, where as $t - b - \tau$ Yukawa unification with $h_t = h_b = h_\tau$ is achieved for larger values of tan $\beta \simeq 60$ at the SUSY GUT scale ($M_U \simeq 2 \times 10^{16}$ GeV).

In a recent analysis, it has been shown that $t - b - \tau$ Yukawa unification for larger tan β($b - \tau$ Yukawa unification for smaller tan β) occurs in nonSUSY Model-(B), but with two light Higgs doublets in the SM below the G_{224}-breaking scale [5]. Detail of solutions of RGEs are given in Ref. [5]. In order to achieve lower values of G_{224}-breaking scale, significant corrections due to threshold and gravitational effects have been noted to exist in this model.

6. Summary and conclusion

Nonsupersymmetric grand unified theories appear to be consistent with the CERN-LEP data and experimental lower limit on proton lifetime with attractive values for neutrino masses and mixings having potentialities to explain oscillations of neutrions. Whereas a class of models with Higgs scalars in the grand desert require a number of additional finetuning of parameters, an alternative model exists with a single scalar $\xi(3, 0, 8)$ near $M_\xi \simeq 10^{10} - 10^{12}$ GeV, where only one additional finetuning may be needed. The other class of models possess left-right gauge groups with or without parity down to the intermediate scales in GUTs like $SO(10)$, $SO(18)$, $SU(16)$, and E_6 *etc.* which do not need any additional finetuning. Although some investigations have been carried out to explain solar neutrino oscillations in some of the models [22], the full potentialities to account neutrino masses and mixings are yet to be analysed. Radiative corrections to see-saw formulas causes significant increase in ν_τ-mass predictions at low energies due to large Yukawa coupling of the top-quark [23]. An interesting feature of these models is that, with or without additional symmetries, they can accommodate degenerate neutrino masses [28]. Yukawa unification for the third generation fermions have also been noted in non SUSY $SO(10)$ at the G_{224}-breaking scale leading to the two-Higgs doublet standard model [5]. In all GUTs with

G_{224P}-intermediate gauge symmetry, the predictions on $\sin^2\theta w(Mz)$ and intermediate scale (M_I) are much more precise compared to other models with a different intermediate symmetry. In conclusion, we observe that it is difficult to rule out non SUSY GUTs on the basis of the available experimental data unless signatures of superpartners are testified in future.

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